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PP 207-214

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Section 35: z Test for One Sample

The national mean for the population who takes the College Board's *SAT-Verbal* is 500.00 and the standard deviation is 100.00. Using the symbols for the population mean and standard deviation, we can say that:

$\mu = 500.00$ and $\sigma = 100.00$

(Note that μ is the symbol for the population mean and σ is the symbol for the population standard deviation.)

Suppose that researchers for the Department of Education of the State of Disrepair (the 51st state admitted to the Union) suspected that their students, on the average, perform more poorly than the national population. (This is their *research hypothesis*.) Suppose they drew a random sample of 200 students who took the test and found that:

$m = 485.00$ and $s = 101.00$

At first glance, the data seem to support their *research hypothesis*; on the average, the sample of students from the state is 15 points below the national population. However, the *null hypothesis* also offers an explanation for the 15-point difference. It states that the difference was created by sampling errors due to the random sampling—that, in fact, the *true* difference is zero. (It asserts that this true difference of zero would have been obtained if the entire population, instead of just a random sample, had been studied.) Thus, we have one difference of 15 points for which we have two explanations:

1. the *research hypothesis* and
2. the *null hypothesis*.

To determine whether the *null hypothesis* is viable, we can test it with a ***z test***.

You probably recall that a z-score for an individual is computed using this formula:[1]

$$z = \frac{X - m}{s}$$

[1]See Section 16 to review z-scores.

It indicates how many standard deviations a person is from the mean of his or her group; z-scores of greater than 1.96 and less than –1.96 occur less than 5% of the time in a normal distribution. Thus, we can say that the probability of drawing a person on a single random draw who has a z-score this extreme is an unlikely event.[2] We can use the same logic (but a modified formula) to determine whether the sample mean of 485.00 is an unlikely event; that is, it is unlikely to be obtained by random sampling from a population with a mean of 500.00. The formula we use for the z test is:

$$z = \frac{\overset{\text{mean}}{m} - \overset{\text{pop. mean}}{\mu}}{SE_m} \text{ (standard error of mean)}$$

The denominator of the formula should look familiar. It is the symbol for the standard error of the mean, which we examined in Section 31. Because we know the standard deviation of the population, we use it to calculate SE_m instead of the standard deviation of the sample because the standard deviation of the population is a value that we know is free of sampling errors while the standard deviation of the sample is subject to such errors. Thus, we first calculate:

$$SE_m = \frac{\overset{\text{SD}}{\sigma}}{\underset{\text{\# of cases/subjects}}{\sqrt{n}}} = \frac{100.00}{\sqrt{200}} = \frac{100.00}{14.142} = 7.071$$

Substituting, we obtain:

$$z = \frac{485.00 - 500.00}{7.071} = \frac{-15.00}{7.071} = -2.121$$

Now that we have the value of z for this z test, we evaluate it to determine if it is an unlikely event. If we determine that a mean of 485.00 is unlikely to be obtained by random sampling from a population with a mean of 500.00, we

[2] See Section 30 to review this concept.

will *reject the null hypothesis* and declare the difference to be *statistically significant*.

Remember that the null hypothesis says that the mean difference of 15 points is merely due to random sampling errors. If we determine that this is unlikely, we will reject the hypothesis. In statistics, as in everyday life, if something is unlikely to be true, we reject it and act as though it is false.

To evaluate our z of -2.121, we will first use the constants 1.96 and -1.96. The table of the normal curve (see Section 30 and Table 1 near the end of this book) tells us that the probability of obtaining a z this extreme is .05 or 5 in 100. Because we obtained a z of -2.121, the odds of obtaining our particular result are *less than* 5 in 100. This result may be reported in one of two ways. Note that they both have the same meaning and implications:

1. The null hypothesis has been rejected at the .05 level.
2. The difference is statistically significant at the .05 level.

We can also evaluate our value of z using the constants of 2.58 and -2.58. As you learned in Section 30, the odds of obtaining a z this extreme are .01 or 1 in 100. Because we obtained a z of -2.121, our result is *not* sufficiently extreme to classify this as an unlikely event at the .01 level. Thus, using this level, we report that:

1. The null hypothesis has *not* been rejected at the .01 level. (Another way of saying this is: We have failed to reject the null hypothesis at the .01 level.)
2. The difference is *not* significant at the .01 level. (Another way of saying this is: The difference is insignificant at the .01 level.)

Notice that the two results we have just examined are not contradictions. In practice, before examining the data, you should select a level (usually .05 or .01) that you will use in your significance test. Had you chosen the .05 level, you would report to your audience that the difference is significant at that level. Had you initially chosen the .01 level, you would report that it is *not* significant at that level. Initially, you should choose only one level.

It is important to note that the decision to *not reject* the null hypothesis is *not* equivalent to *accepting* the null hypothesis. Recall at the beginning of this section that we had two hypotheses that might explain the difference—the null hypothesis and the research hypothesis. If we fail to reject the null hypothesis, we are still left with two hypotheses—the null hypothesis, which we have failed to reject, and the research hypothesis, which cannot be directly tested with statistics. Tests exist only for the null hypothesis. Thus, if we fail to reject the null hypothesis, we have an inconclusive result because there are two hypotheses that explain the difference.

Let us review the ***decision rules*** for the *z* test we have just considered:

1. If the value of *z* that you computed is as extreme as:[3]

   1.96 or –1.96, declare the difference to be significant at the .05 level (i.e., reject the null hypothesis).

   2.58 or –2.58, declare the difference to be significant at the .01 level (i.e., reject the null hypothesis).

2. If the value of *z* that you computed does not meet the first condition, do *not* declare the difference to be significant and do *not* reject the null hypothesis.

Let us apply the decision rules to several examples. In each example, a random sample has been drawn and tested. In each case, the population mean and standard deviation are known:

Example 1:

Researcher Smith obtained a value of *z* of 3.458 for the difference between two means. She chose the .05 level before starting her study.

[3]Select one of the two rules *before* examining the data.

1. Should she reject the null hypothesis?

    Yes, because 3.458 is more extreme than 1.96.

2. Should she declare the difference to be statistically significant?

    Yes.

Example 2:

Researcher Doe obtained a value of z of 1.786 for the difference between two means. He chose the .05 level before starting his study.

1. Should he reject the null hypothesis?

    No, because 1.786 is not as extreme as 1.96.

2. Should he declare the difference to be statistically significant?

    No.

Example 3:

Researcher Jones obtained a value of z of 2.966 for the difference between two means. She chose the .01 level before starting her study.

1. Should she reject the null hypothesis?

    Yes, because 2.966 is more extreme than 2.58.

2. Should she declare the difference to be statistically significant?

    Yes.

Example 4:

Researcher Daly obtained a value of z of –1.999 for the difference between two means. He chose the .01 level before starting his study.

1. Should he reject the null hypothesis?

    No, because –1.999 is not as extreme as –2.58.

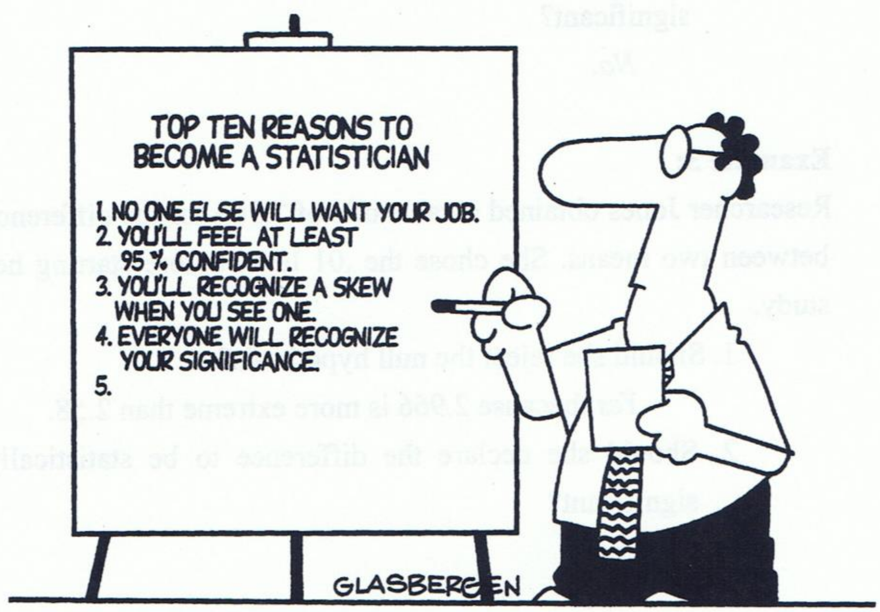
2. Should he declare the difference to be statistically significant?

   No.

The *decision rules* that you have learned about in this section apply to what are known as *two-tailed tests*. These are usually appropriate and, thus, widely used. *Two-tailed tests* and *one-tailed tests* are defined and compared in the next section.

Terms to Review Before Attempting Worksheet 35

z test, decision rules



Worksheet 35: z Test for One Sample

> ***Riddle*: According to Voltaire, what does the length of an argument tell us about who is right?**

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. If the standard deviation of a population is 50.00 and a sample of 42 subjects is drawn at random, what is the value of the standard error of the mean?

$SE_m = \frac{\sigma}{\sqrt{n}} \quad = \frac{50}{\sqrt{42}} = \frac{50}{6.481} = 7.715$

2. In a study, the mean of a population equals 45.00, the mean of a random sample equals 48.00, and the standard error of the mean equals 2.540. What is the value of z?

$z = \frac{m - \mu}{SE_m} \quad \frac{48.00 - 45.00}{2.540} = \frac{3}{2.540} \quad z = 1.181$

3. In a study, the mean of a population equals 100.00, the mean of a random sample equals 90.00, and the standard error of the mean equals 4.110. What is the value of z?

$z = \frac{m - \mu}{SE_m} = \frac{90 - 100}{4.110} = \frac{-10}{4.110} \quad z = -2.433$

4. What constants are used to determine if a z is an unlikely event at the .05 level?

1.96 + −1.96

5. What constants are used to determine if a result is significant at the .01 level?

2.58 + −2.58

Worksheet 35 (Continued)

6. For the type of study described in this section, should the null hypothesis be rejected at the .05 level if z equals 2.343?

   yes more than 1.96

7. For the type of study described in this section, should the null hypothesis be rejected at the .05 level if z equals 1.74?

   No less than 1.96

8. "For the type of study described in this section, a value of z of 2.997 means that the null hypothesis should be rejected at the .01 level." Is this statement true or false?

   True b/c more than 2.58

9. "*Not rejecting the null hypothesis* is equivalent to *accepting the null hypothesis*." Is this statement true or false?

   False