7.11.09

Z

PP

Section 30: Probability and the Normal Curve

In the empirical approach to knowledge, we make observations and, based on them, make decisions. Based on previous observations, we can establish probabilities regarding the occurrence of specific events in the future. Weather forecasting is based on this approach. Events such as high and low pressure systems are observed, and predictions are made based on previous observations of their effects on the weather.

Fortunately, for many problems, empirical probabilities are easy to determine because many distributions are normal.[1] Suppose, for example, that we conducted a large national survey to determine knowledge of basic math skills; each subject was administered a basic math test. If we found that the distribution was normal and that the mean was 50.00 and the standard deviation was 7.00, we could use this information to establish probabilities. To do so, we would need to use z-scores. You may recall that the formula for them is:[2]

$$z = \frac{X - M}{S}$$

In this example, what is the probability of drawing an individual at random from the population who has a score of 64 or higher? To answer the question, first calculate the corresponding z-score:

$$z = \frac{64 - 50.00}{7.00} = \frac{14.00}{7.00} = 2.00$$

Then look up the z-score in Table 1 near the end of this book. There we find that the percent of cases in the smaller part (Column 4) is 2.28%. Divide this by 100, and we obtain a probability of .0228, which indicates that there is only slightly more than 2 chances in 100 of drawing such a person from the population. This is referred to as a ***one-tailed probability*** because we asked the

[1]The normal curve was first introduced in Section 9 and explored in Sections 13 through 16.
[2]See Section 16 to review z-scores.

question about only the upper tail of the normal distribution—the right-hand tail of the distribution in Figure 30.1.

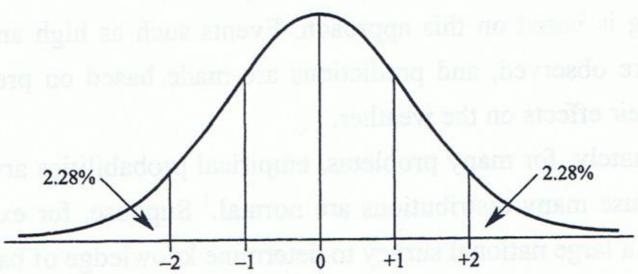

Figure 30.1. Normal distribution with selected *z*-scores.

Suppose, instead, we asked this question: What is the probability of drawing at random an individual with a *z*-score as high as 2.00 (or higher) *or* as low as –2.00 (or lower)? Obviously, the odds of doing so are double those of drawing just one of these. Thus, the odds are 2 × .0228 = .0456, which is just a little more than 4 in 100. This is called a ***two-tailed probability*** because we are asking about the odds of drawing an individual at either tail of the normal distribution in Figure 30.1. The importance of distinguishing between one-tailed and two-tailed probabilities will become clear in later sections.

The probabilities for both events described above are ***unlikely events***. In most sciences, conventional wisdom indicates that any event that has a probability of occurrence of .05 or less is usually classified as unlikely to occur at random. You will notice in Table 1, under *Values of Special Interest*, that a *z*-score of 1.96 has only a 2.5% chance of occurrence as a one-tailed probability (see Column 4); as a two-tailed probability, it has a 5.0% chance of occurrence (2 × 2.5% = 5.0%). Thus, an event with a *z*-score of 1.96 or greater or –1.96 or less (such as 1.97 or –1.97) is classified as an unlikely event.

Table 1 also indicates that a *z*-score of 2.58 or higher has only a .49% (or almost 1/2 of 1%) chance of occurrence. The corresponding two-tailed probability is .98% or almost 1% for *z*-scores of 2.58 or –2.58. Some scientists *only* classify an event as unlikely if its likelihood is 1% or less.

There is no rule of nature that says at what point an event should be classified as unlikely. However, the 5% and 1% guidelines have evolved over time as the two most widely used. The only universally accepted rule is that a researcher must decide *in advance* of examining the data what guideline will be followed for declaring an event to be unlikely. Theoretically, any percentage may be specified, but, to be accepted in most scientific circles, 5% or lower is generally used.

Identifying unlikely events is the basis of many of the tests of statistical significance presented in later sections.

Terms to Review Before Attempting Worksheet 30

one-tailed probability, two-tailed probability, unlikely events

"I have a photographic memory— I just seem to be out of film today."

Worksheet 30: Probability and the Normal Curve

> ***Riddle*: What did George Washington do when he was asked for his ID?**

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. What is the one-tailed probability of drawing a subject with a z-score of 1.35 or higher at random from a normal distribution? (Hint: Remember to divide the percentage you obtain from Table 1 by 100.)

   .09 OR .0885

2. What is the one-tailed probability of drawing a subject with a z-score of –1.70 or lower at random from a normal distribution?

   .04457 OR .04

3. What is the two-tailed probability of drawing a subject with a z-score as extreme as 1.80 or –1.80 at random from a normal distribution?

   $2 \times .03593 = .07186$

   (.07 – .07)

4. For a normal distribution with a mean of 100.00 and a standard deviation of 16.00, what is the one-tailed probability of drawing a subject with a score of 124 or greater at random from a normal distribution?

   $Z = \frac{X - M}{S}$ $\quad \frac{124 - 100}{16} = \frac{24}{16} = 1.5$

   $= .067$

   .07

Worksheet 30 (Continued)

5. For a normal distribution with a mean of 40.00 and a standard deviation of 8.00, what is the one-tailed probability of drawing a subject with a score of 30 ~~or less at~~ random from a normal distribution?

$Z = \frac{X-M}{S} \quad \frac{30-40}{8} \quad \frac{-10}{8} = -1.25$ .10564 .11

6. Using the information in Question 5, what is the probability of drawing a subject with a score as extreme as 30 or 50 at random from a normal distribution?

? 0 (2-tail) (.22) $\frac{50-40}{8} = \frac{10}{8} = 1.25$ .10564 .11

7. Using the 5% guideline, should the answer to Question 2 be classified as an unlikely event?

yes

8. Using the 1% guideline, should the answer to Question 5 be classified as an unlikely event?

? 6 No

Solution section: