7.12.09

PP

Section 31: Standard Error of the Mean

Suppose that there is a large population with a mean of 100.00 and a standard deviation of 16.00 on a standardized test. Further, suppose that we do not have this information but wish to estimate the mean and standard deviation by testing a sample. When we draw a random sample and administer the test to just the sample, will we correctly estimate the population mean as 100.00? The answer is probably not. Remember that random sampling introduces random or chance errors—known as sampling errors.

At first, this situation may seem rather hopeless, but we have the advantage of using an unbiased, random sample—meaning that no factors are systematically pushing our estimate in the wrong direction. In such a situation, using a large sample increases the likelihood that we are correct or, at least, not likely to make a large error.[1]

We also have the advantage of the ***central limit theorem***. To understand this theorem, you must first understand the sampling distribution of means. Suppose that we drew not just one sample but many samples at random. That is, we drew a sample of 60, tested the subjects, and computed the mean; then drew another sample of 60, tested the subjects, and computed the mean; then drew another sample of 60, tested the subjects, and computed the mean; etc. We would then have a very large number of means—known as the ***sampling distribution of means.*** *The central limit theorem says that the distribution of these means is normal in shape*. The normal shape will emerge even if the underlying distribution is skewed, provided that the sample size is reasonably large (about 60 or more). The mean of an indefinitely large sampling distribution of means will equal the population mean. The standard deviation of the sampling distribution is known as the ***standard error of the mean***. Keep in mind that the means vary from each other only because of chance errors created by random sampling. That is, we are drawing random samples from the same population and administering the same test over and over, so all of the means should have the same value except for the effects of random errors.

[1]Review Sections 27 and 28 on sampling.

Therefore, there is variation among the means only because of sampling errors. For this reason, the *standard deviation of the sampling distribution* is known as the *standard error of the mean.*

In practice, we usually draw a single sample, test it, and calculate its mean and standard deviation.[2] Therefore, we are not certain of the value of the population mean nor do we know the value of the standard error of the mean that we would obtain if we had sampled repeatedly. Fortunately, we do know two very useful things:

➔ The larger the sample, the smaller the standard error of the mean.

➔ The less the variability in the population, the smaller the standard error of the mean. For example, consider a population in which there is no variability—that is, in which all subjects are identical. In this case, the standard error of the mean (i.e., the standard deviation of the sampling distributions of means) equals 0.00 (i.e., all the means will be identical and their standard deviation will be zero). In practice, we cannot be certain how much variability there is in a population from which we have only sampled. However, we can use the standard deviation of the sample that we have drawn as an estimate of the amount of variability in the population; for example, if we observed a very small standard deviation for a random sample, it would be reasonable to guess that the population has relatively little variation.

Given these two facts and some statistical theory that is not covered here, statisticians have developed this formula for estimating the standard error of

[2]Use the second formula in Appendix A when estimating the standard deviation of a population from a sample.

the mean based only on the information we have about a given random sample from a population:

$$SE_m = \frac{s}{\sqrt{n}}$$

Let's apply the formula in three examples to see how it works:

Example 1:

For a randomly selected sample, $m = 75.00$, $s = 16.00$, and $n = 64$. If we divide 16.00 by the square root of 64 (i.e., 8), we estimate that the standard error of the mean equals 2.00. This is an estimate of a ***margin of error*** that we should keep in mind when interpreting the sample mean of 75.00.

Keep in mind, too, that the *standard error of the mean* is an estimate of the *standard deviation of the sampling distribution of the means*, which is normal in shape when the sample size is relatively large. You may recall from your study of the standard deviation that about 68% of the cases lie within one standard deviation unit of the mean. Thus, we would expect about 68% of all sample means to lie within 2.00 points of the true (or population) mean. If we use the mean of 75.00, which was actually obtained, as an estimate of the population mean based on a random sample, we could estimate that odds are 68 out of 100 that the population mean lies between 73.00 (75.00 – 2.00 = 73.00) and 77.00 (75.00 + 2.00 = 77.00). The values of 73.00 and 77.00 are known as the ***limits of the 68% confidence interval for the mean***. That is, we have about 68% confidence that the true mean lies between 73.00 and 77.00.

Example 2:

For a randomly selected sample, $m = 75.00$, $s = 16.00$, and $n = 128$. If we divide 16.00 by the square root of 128 (i.e., 11.314), we estimate that the standard error of the mean equals 1.41. Notice that this is substantially smaller than the standard error we obtained in Example 1. This is because the sample size is twice that in Example 1. However, also notice that the standard error has not been cut in half. This is because we are dividing by the *square root* of n.[3]

The limits of the 68% confidence interval for Example 2 are 73.59 (75.00 – 1.41 = 73.59) and 76.41 (75 + 1.41 = 76.41). The larger sample size in Example 2 has given us a smaller confidence interval than in Example 1.

Example 3:

For a randomly selected sample, m = 75.00, s = 5.00, and n = 128. If we divide 5.00 by the square root of 128 (i.e., 11.314), we estimate that the standard error of the mean equals 0.44. The limits of the 68% confidence interval are 74.56 and 75.44. The smaller standard deviation in Example 3 has given us a smaller confidence interval than in Example 2.

It should be obvious that a small confidence interval is desirable because it indicates that the sample mean is probably close to the true mean. Of the two variables that affect the size of the standard error of the mean—the sample size and the variability of the sample—we often have direct control of the sample size. By using reasonably large samples, we can minimize the standard error of the mean.

Of course, 68% confidence is far short of certainty. The next section covers how to build 95% and 99% confidence intervals—intervals within which we can have much greater confidence that the population mean lies.[4]

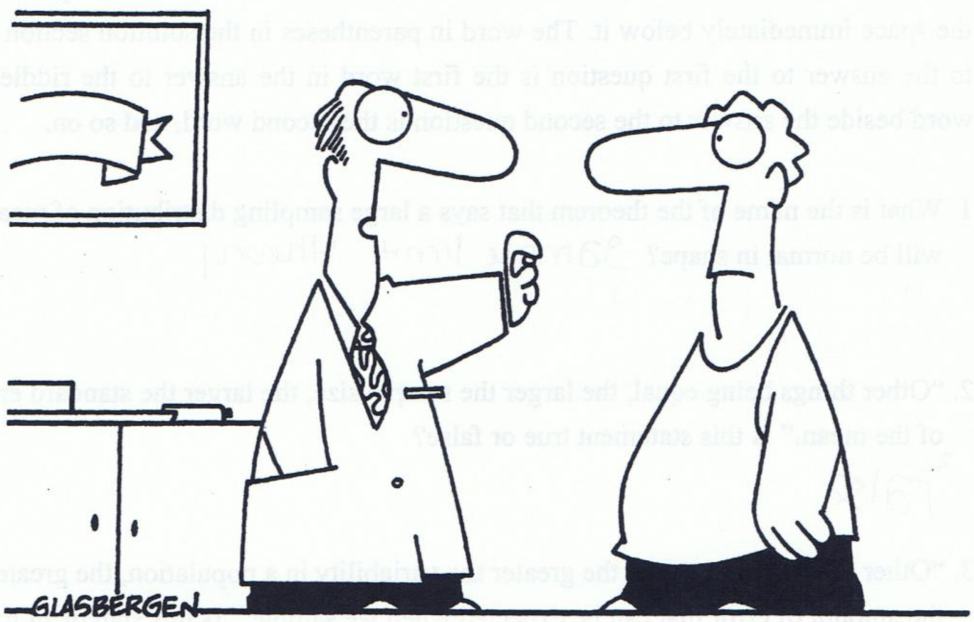
It is important to keep in mind that the confidence limits are only valid if we analyze the results obtained with unbiased (random) sampling. Each bias has its own unique and usually unknown effects on the results, and there are no generalizable techniques for estimating the amount of error created by them.

[3]You may recall from Section 27 that increasing the sample size produces diminishing returns, which is clearly illustrated here.

[4]Appendix F presents the formulas for calculating the standard error of a median and the standard error of a percentage.

Terms to Review Before Attempting Worksheet 31

central limit theorem, sampling distribution of means, standard error of the mean, margin of error, limits of the 68% confidence interval for the mean



"Statistics show that to prevent a heart attack, you should take one aspirin every day. Take it out for a jog, then take it to the gym, then take it for a bike ride...."

Worksheet 31: Standard Error of the Mean

Riddle: According to Frank Lloyd Wright, how are the truth and the facts related?

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. What is the name of the theorem that says a large sampling distribution of means will be normal in shape? Sample limit theory

2. "Other things being equal, the larger the sample size, the larger the standard error of the mean." Is this statement true or false?

False

3. "Other things being equal, the greater the variability in a population, the greater the amount of error that can be expected when we sample." Is this statement true or false?

true

4. If we double the size of a sample, can we expect to have half the amount of error due to random sampling?

NO

5. For a randomly selected sample, $m = 50.00$, $s = 10.00$, and $n = 64$. What is the value of the standard error of the mean?

$SE_m = \frac{S}{\sqrt{n}} = \frac{10}{\sqrt{64}} = \frac{10}{2}$

$SE_m = 1.25$

Worksheet 31 (Continued)

6. For a randomly selected sample, $m = 50.00$, $s = 10.00$, and $n = 144$. What is the value of the standard error of the mean?

   $SE_m = \frac{s}{\sqrt{n}} = \frac{10}{\sqrt{144}} = \frac{10}{12}$ $SE_m = .83$

7. For a randomly selected sample, $m = 100.00$, $s = 16.00$, and $n = 100$. What are the limits of the 68% confidence interval for the mean?

   $SE_m = \frac{s}{\sqrt{}} = \frac{16}{\sqrt{100}} = \frac{16}{10}$ $SE_m = 1.6$

   98.4 – 101.6

8. For a randomly selected sample, the mean equals 90.00 and the standard error of the mean equals 5.22. What are the limits of the 68% confidence interval for the mean?

   84.78 – 95.22