7.11.09

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Section 14: A Closer Look at the Standard Deviation

In the previous section, you learned that the ***standard deviation*** is a measure of how much scores vary from their mean. More specifically, you learned that about 68% of the scores in a normal distribution lie within one standard deviation unit of the mean. In this section, you'll learn some additional rules for interpreting the standard deviation.[1]

The ***approximate 95% rule*** says that if you go out 2 standard deviation units on both sides of the mean in a normal distribution, you will find approximately 95% of the cases. Here's an example of the application of the approximate 95% rule:

Example 1:
The mean for a group equals 35.00 and the standard deviation equals 6.00. Two standard deviation units equal 12.00 points (2 × 6.00 = 12.00). Thus, if you (a) go up 12 points from the mean (35.00 + 12.00 = 47.00) and (b) go down 12 points from the mean (35.00 – 12.00 = 23.00), you have identified the scores (47.00 and 23.00) between which approximately 95% of the cases lie.

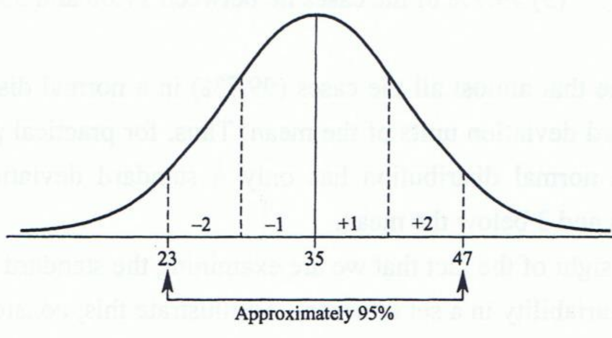

Figure 14.1. Normal curve illustrating 95% rule.

[1]The rules are derived from the table of the normal curve, which is introduced in a later section.

The ***99.7% rule*** says that if we go up and down 3 standard deviation units from the mean, we will find 99.7% of the cases. For the information in Example 1, multiply 3 times the standard deviation ($3 \times 6.00 = 18.00$). Going up and down 18 points from the mean yields these scores: 53.00 and 17.00. This is illustrated here:

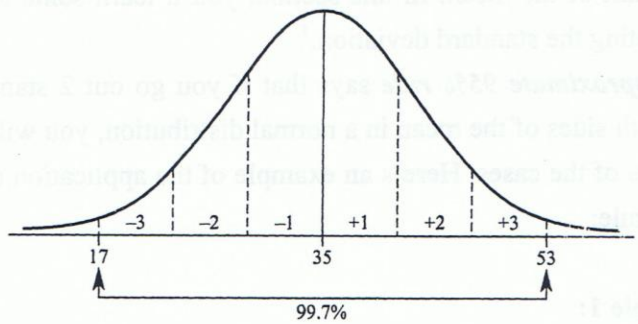

Figure 14.2. Normal curve illustrating 99.7% rule.

Let's review:

If $M = 35.00$ and $S = 6.00$, then:

(1) 68% of the cases lie between 29.00 and 41.00;

(2) 95% of the cases lie between 23.00 and 47.00;

(3) 99.7% of the cases lie between 17.00 and 53.00.

You can see that almost all the cases (99.7%) in a normal distribution lie within 3 standard deviation units of the mean. Thus, for practical purposes we can say that a normal distribution has only 6 standard deviation units—3 above the mean and 3 below the mean.

Don't lose sight of the fact that we are examining the standard deviation to determine the variability in a set of scores. To illustrate this, consider Example 2. The mean is the same as in Example 1, but the standard deviation is only half its size. Results of applying the three rules are shown. By comparing the two examples, it becomes obvious that the smaller the standard deviation, the less far out you need to go to capture a given percentage of cases. In fact, with a standard deviation of only 3.00, you need to go only to 26.00 and 44.00 to capture 99.7% of the cases.

Example 2:

When $M = 35.00$ and $S = 3.00$, then approximately:

(1) 68% of the cases lie between 32.00 and 38.00;

(2) 95% of the cases lie between 29.00 and 41.00;

(3) 99.7% of the cases lie between 26.00 and 44.00.

If you were reporting on research in which the groups in Example 1 and Example 2 were being compared, you could say that the two groups are equal on the average (as measured by the mean) but that the first group has twice the variability (as measured by the standard deviation) than the second group. Potentially, this could be very important information for your readers.

When doing the following worksheet, keep the following multipliers in mind. Multiply the standard deviation for a set of scores by the appropriate multiplier before adding and subtracting from the mean.

Rule	Multiplier
68%	1
95%	2
99.7%	3

Additional rules are discussed in the next section. Remember, all of these rules strictly apply only in the case of a normal distribution.

Terms to Review Before Attempting Worksheet 14

approximate 95% rule, 99.7% rule

Worksheet 14: A Closer Look at the Standard Deviation

> ***Riddle*: When do most people stop believing in heredity?**

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. If you go out 1 standard deviation unit on both sides of the mean in a normal distribution, what percentage of the cases will you capture?

   68%

2. If you go out 3 standard deviation units on both sides of the mean in a normal distribution, what percentage of the cases will you capture?

   99.7%

3. For $M = 100.00$ and $S = 10.00$, approximately what percentage of the cases lie between 80.00 and 120.00?

   95%

4. For $M = 55.00$ and $S = 7.00$, between what two values do approximately 95% of the cases lie in a normal distribution?

   $55 - 14$ $\quad 55 + 14$ $\qquad 41 \sim 69$

5. For $M = 90.00$ and $S = 15.00$, between what two values do 99.7% of the cases lie in a normal distribution?

   $90 - 45 = 45$ $\quad 90 + 45 = 135$ $\qquad 45 \sim 135$

Worksheet 14 (Continued)

6. What is the multiplier for the approximate 95% rule?

2

7. What is the multiplier for the 99.7% rule?

3

8. For all practical purposes, the normal curve has how many standard deviation units?

6

Section 15: Another Look at the Standard Deviation

When the standard deviation is reported in conjunction with the mean of a set of scores, it gives an indication of the amount of variability in the distribution. For example, suppose you read the following statement in a report:

> For the population of Martians, the mean score on the Human Awareness Scale is 44.00 and the standard deviation is 4.00. The distribution is normal.

From this, you should be able to picture in your mind's eye that about 68% of the Martians had scores between 40.00 and 48.00.[1] Put another way, the vast majority of the Martians had scores within 4 points of the mean. This indicates the amount of variability because it indicates the number of points within which the vast majority fall.

In Section 14, you learned that *approximately* 95% of the cases lie within 2 standard deviation units of the mean. To be more precise, the ***precise 95% rule*** says that if you go out 1.96 standard deviation units from the mean in a normal distribution, you will find 95% of the cases.[2] The constant 1.96 is derived from the definition of the normal curve.[3] To some students, at first this seems a little like magic. It might help to keep this in mind: If 95% of the cases do *not* lie within 1.96 standard deviation units of the mean, the distribution is *not* normal. Thus, this rule applies to all normal distributions, regardless of the value of their means and standard deviations.

Let's apply the precise 95% rule to the Martian example:

1. Multiply 1.96 times S ($1.96 \times 4.00 = 7.84$).
2. Subtract the result of Step 1 from the mean ($44.00 - 7.84 = 36.16$).
3. Add the result of Step 1 to the mean ($44.00 + 7.84 = 51.84$).

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[1]To review this rule, see Section 13.
[2]Note that 1.96 is very close to 2, which led to the approximate rule.
[3]The table of the normal curve is introduced in Section 18. When we examine it, you will again encounter the constant 1.96.

The results of Steps 2 and 3 yield the points between which 95% of the cases lie; thus, 95% of the cases in this normal distribution lie between 36.16 and 51.84.

The ***99% rule*** says that if you go out 2.58 standard deviation units from the mean, you will identify the values between which 99% of the cases lie.[4] Let's apply the 99% rule to the Martian example:

1. Multiply 2.58 times S ($2.58 \times 4.00 = 10.32$).
2. Subtract the result of Step 1 from the mean ($44.00 - 10.32 = 33.68$).
3. Add the result of Step 1 to the mean ($44.00 + 10.32 = 54.32$).

The results of Steps 2 and 3 yield the points between which 99% of the cases lie; thus, 99% of the cases in this normal distribution lie between 33.68 and 54.32.

The standard deviation has been examined in detail not only because it is useful in describing the variability in a group but also because it is used in a variety of other statistical procedures. These procedures will be much easier to master if you feel thoroughly comfortable with the standard deviation.

Terms to Review Before Attempting Worksheet 15

precise 95% rule, 99% rule

1.96 2.58

[4]In the previous section, you learned that if you go out 3 standard deviation units from the mean, you identify the values between which 99.7% of the cases lie. As you will see later in this book, 99% is of more interest than 99.7% for advanced statistical work.

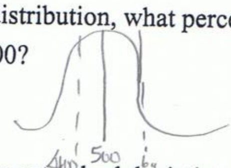
Worksheet 15: Another Look at the Standard Deviation

***Riddle*: What happens if a wealthy person dies without a will?**

DIRECTIONS: To find the answer to the riddle, write the answer to each question in the space immediately below it. The word in parentheses in the solution section next to the answer to the first question is the first word in the answer to the riddle, the word beside the answer to the second question is the second word, and so on.

1. If M = 500.00 and S = 100.00 in a normal distribution, what percentage of the subjects have scores between 400.00 and 600.00?

68%



2. According to the precise 95% rule, how many standard deviation units should you go out from the mean in order to capture 95% of a population in a normal distribution?

2 1.96

3. Does the precise 95% rule apply to all normal distributions?

Yes

4. In a normal distribution with M = 78.56 and S = 12.92, between what two values does the middle 95% of a population lie according to the precise rule?

$1.96 \times 12.92 = 25.32$

$53.24 \sim 103.88$

5. In a normal distribution with M = 50.00 and S = 8.00, what percentage of the population lies between 29.36 and 70.64?

99%

$1.96 \times 8 = 15.68$

$2.58 \times 8 = 20.64$

Worksheet 15 (Continued)

6. In a normal distribution with $M = 150.00$ and $S = 20.00$, what percentage of the population lies between 110.80 and 189.20? $1.96 \times 20 = 39.2$

$2.58 \times 20 = 51.6$

95%

7. In a normal distribution with $M = 11.52$ and $S = 1.40$, between what two values does 99% of the population lie? $2.58 \times 1.40 = 3.61$

7.91 ~ 15.13

8. If Group A has $M = 52.39$ ($S = 3.44$) and Group B has $M = 41.55$ ($S = 4.19$), which group has greater variability? Group B